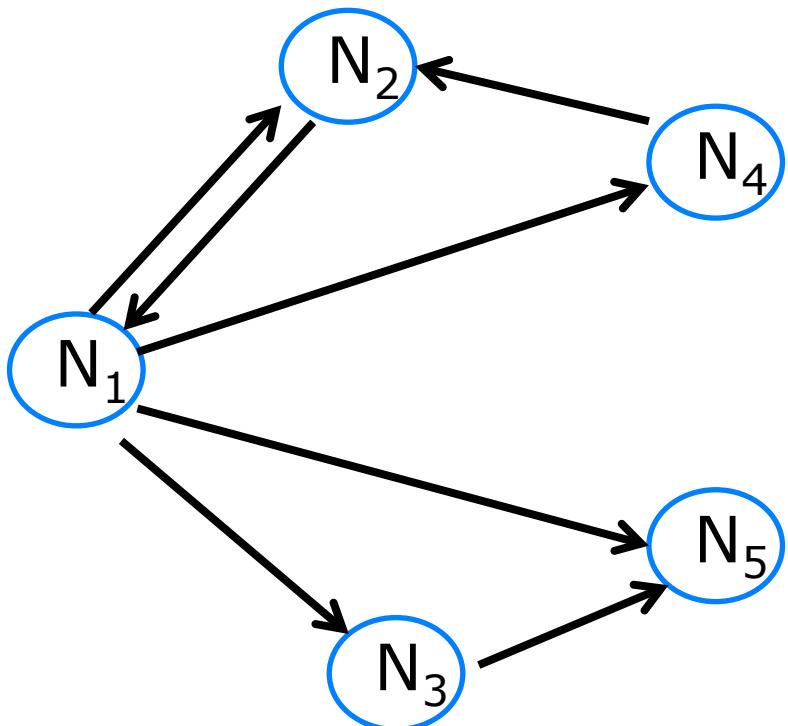


# Graphs and more

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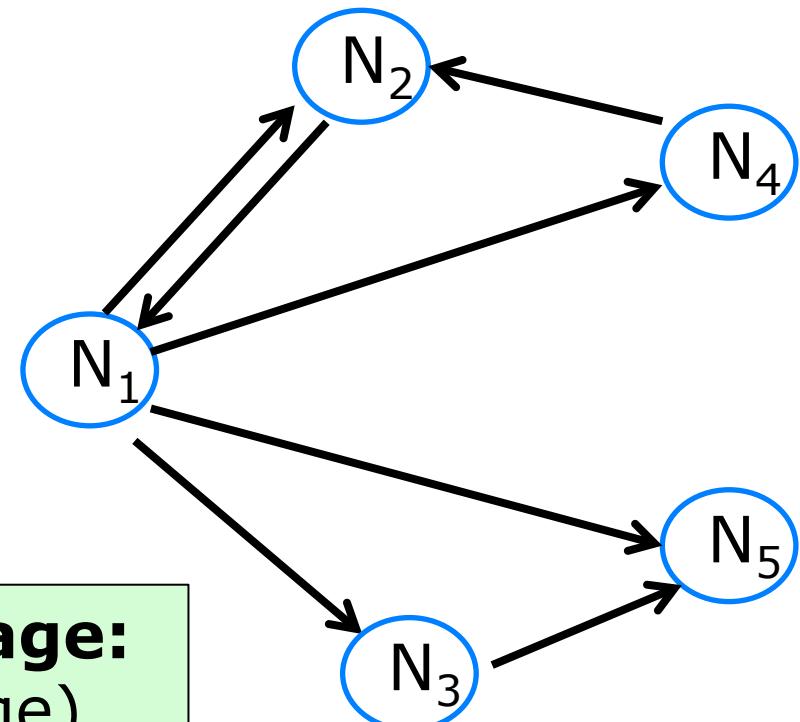
# Directed Graph



- Set of nodes  
 $\{N_1, N_2, N_3, N_4, N_5\}$
- Set of arcs  
 $\{(N_1, N_2), (N_1, N_3), (N_1, N_4), (N_1, N_5), (N_2, N_1), (N_3, N_5), (N_4, N_2)\}$

# Application scenario: web search engine

- Represent the web structure using a directed graph
- Web pages processed concurrently by multiple servers
- Incremental processing

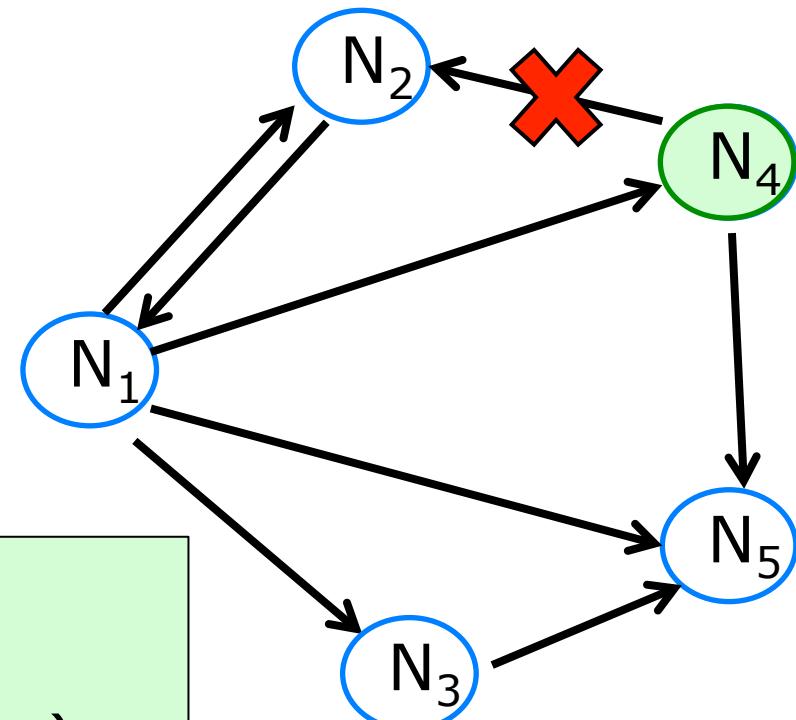


## When processing a web page:

1. Add node (for the web page)
  2. Add arc (for links)
- NOTE: do not add tail node

# Application scenario: web search engine

- Web evolves over time
  - Links change
  - Pages are removed
  - ... and recreated
- Need to update the graph structure

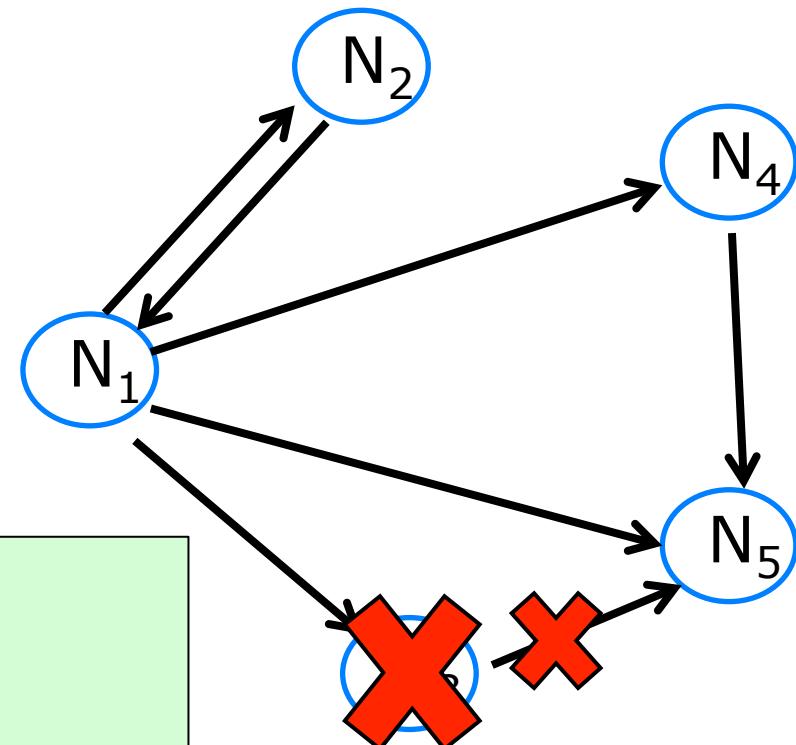


**When re-processing an existing web page:**

1. Add new arcs (for new links)
2. Remove arcs (for removed links)

# Application scenario: web search engine

- Web evolves over time
  - Links change
  - Pages are removed
  - ... and recreated
- Need to update the graph structure

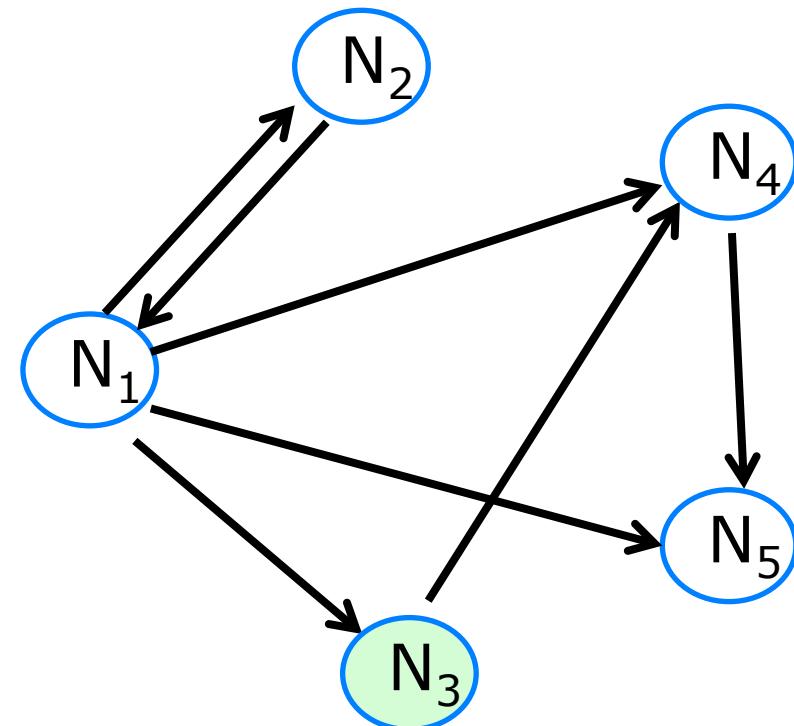


**When re-processing a removed web page:**

1. Remove arcs
2. Remove node

# Application scenario: web search engine

- Web evolves over time
  - Links change
  - Pages are removed
  - ... and recreated
- Need to update the graph structure



# Graph data type

- Two sets: nodes + arcs
- Update operations:
  - `addNode( node)`
  - `removeNode( node)`
  - `addArc( head, tail)`
  - `removeArc( head, tail)`
- Query operations:
  - `lookupNode( node): boolean`
  - `lookupArc( head, tail): boolean`
  - `listNode()`
  - `listArc()`

# Semantics on concurrent operations

- $\text{addNode}(n) \parallel \text{addNode}(n)$ 
  - n is added to the set of nodes
- $\text{removeNode}(n) \parallel \text{removeNode}(n)$ 
  - n is removed to the set of nodes
- $\text{addNode}(n) \parallel \text{removeNode}(n)$ 
  - Alternatives:

1. n added to the set of nodes OR-set

2. n removed from the set of nodes OR-remove-set

3. result depends on (real-time or logical) timestamps LWW-set

4. result depends on the number of times operation  
has been executed counter-set  
(Weiss et. al.)

# Semantics on concurrent operations (cont.)

- $\text{addArc}( n_1, n_2 ) \parallel \text{addArc}( n_1, n_2 )$ 
  - $(n_1, n_2)$  is added to the set of arcs
- $\text{removeArc}( n_1, n_2 ) \parallel \text{removeArc}( n_1, n_2 )$ 
  - $(n_1, n_2)$  is removed to the set of nodes
- $\text{addArc}( n_1, n_2 ) \parallel \text{removeArc}( n_1, n_2 )$ 
  - Alternatives:

1. $(n_1, n_2)$ added to the set of nodes	OR-set
2. $(n_1, n_2)$ removed from the set of nodes	OR-remove-set
3. result depends on (real-time or logical) timestamps	LWW-set
4. result depends on the number of times operation has been executed	counter-set (Weiss et. al.)

# Semantics on concurrent operations (cont.)

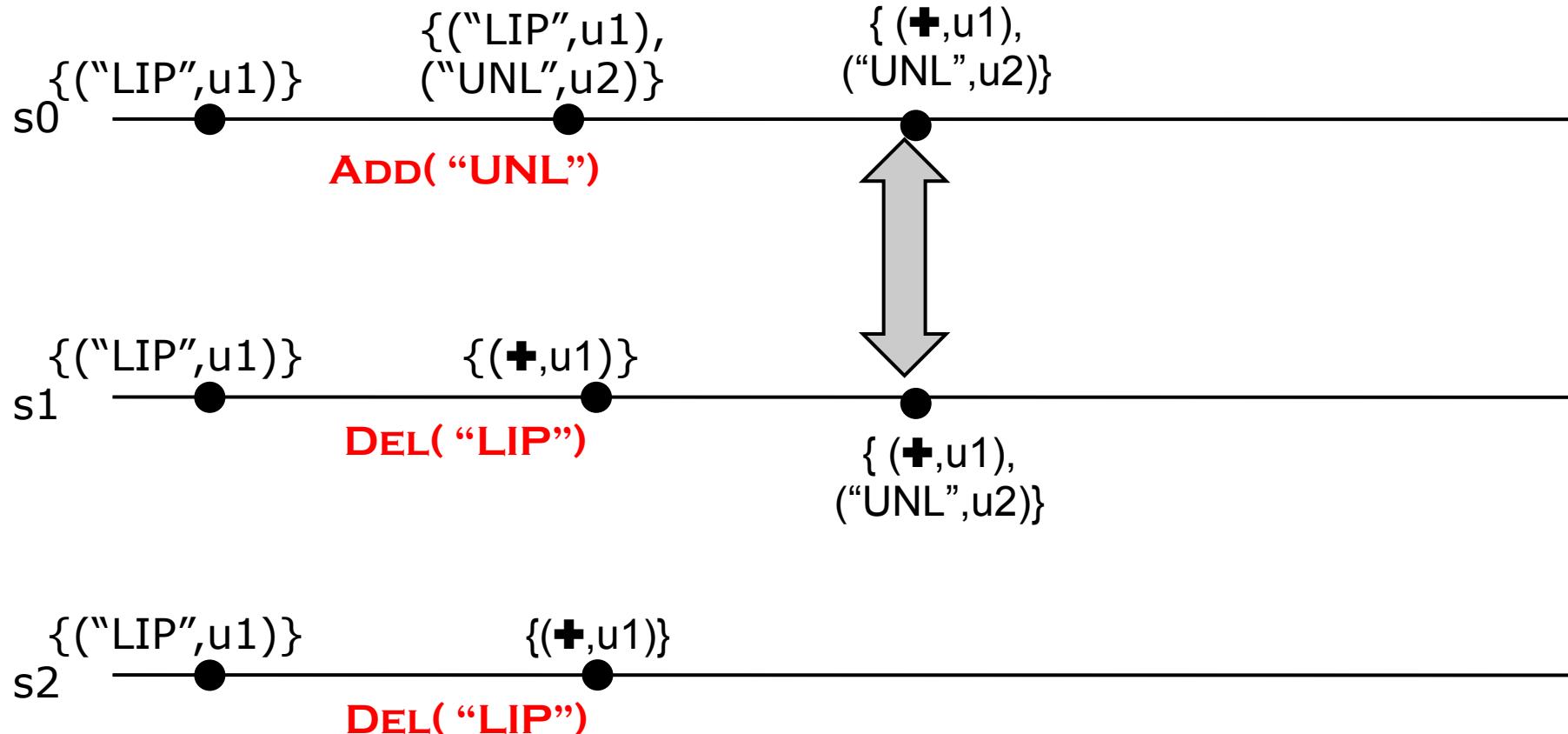
- $\text{addNode}(n) \parallel \text{addArc}(n_1, n_2), n=n_1 \vee n=n_2$ 
  - $n$  added,  $(n_1, n_2)$  added
- $\text{removeNode}(n) \parallel \text{removeArc}(n_1, n_2), n=n_1 \vee n=n_2$ 
  - $n$  removed,  $(n_1, n_2)$  removed
- $\text{addNode}(n) \parallel \text{removeArc}(n_1, n_2), n=n_1 \vee n=n_2$ 
  - $n$  added,  $(n_1, n_2)$  removed
- $\text{removeNode}(n) \parallel \text{addArc}(n_1, n_2), n=n_1 \vee n=n_2$ 
  - $n$  removed,  $(n_1, n_2)$  added but hidden

$\text{lookupArc}(n_1, n_2) : \text{return } (n_1, n_2) \in A \wedge \text{lookup}(n_1) \wedge \text{lookup}(n_2)$

# Observed-remove Set CRDT

- Semantics
  - Maintains a set of atoms (that are not necessarily unique)
  - On concurrent add and delete of the same atom, adds wins
- Operations:
  - Add( a ) -> adds the atom to the set
  - Delete( a ) [Pre:  $a \in \text{set}$ ] -> removes the atom from the set
  - Lookup() -> returns the atoms in the set

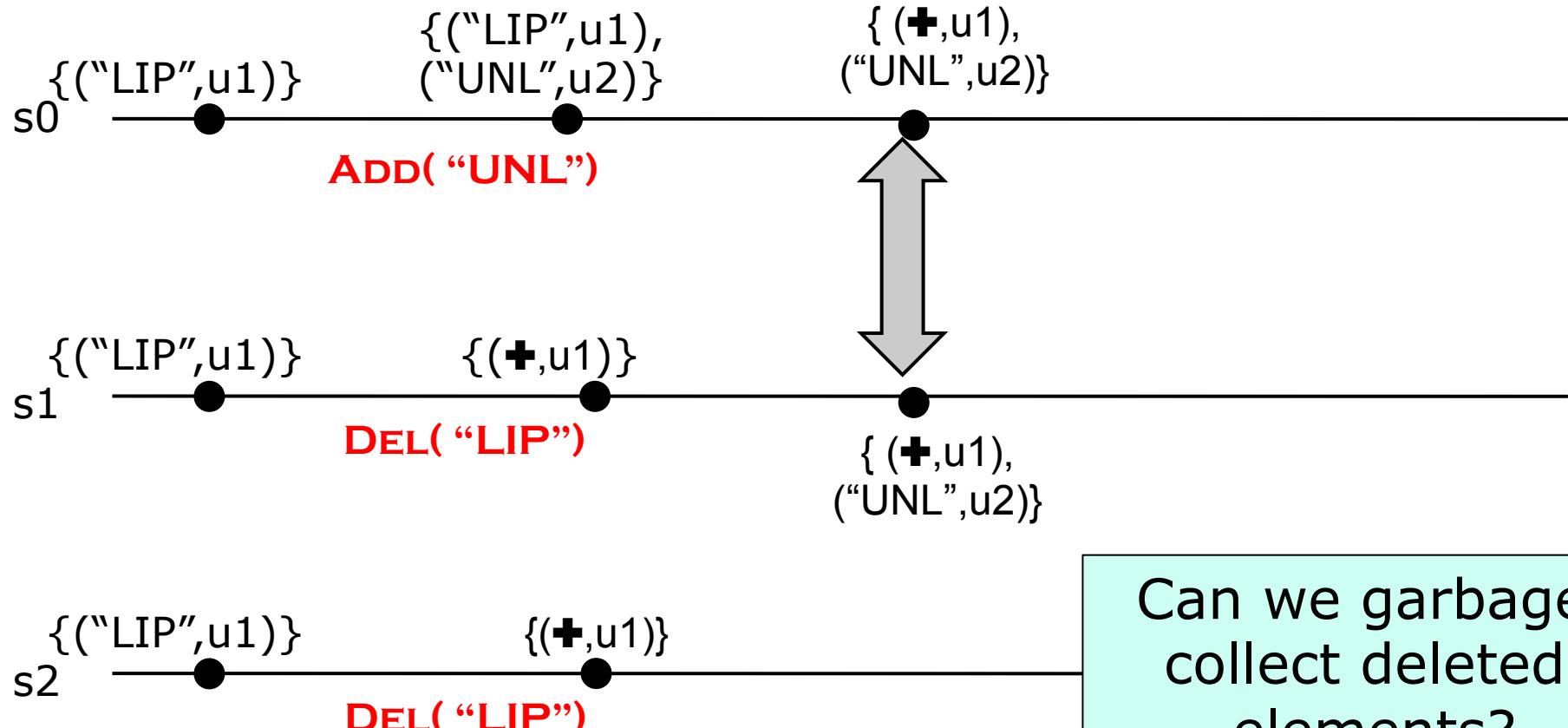
# Observed-remove Set CvRDT



# Observed-remove Set CvRDT

- State,  $S$ 
  - $S.A = \text{set of pairs } (a, \text{uid}_a)$
  - $S.R = \text{set of tombstone uids}$
- Operations:
  - $\text{add } (a) \rightarrow \text{add}( (a, \text{uid}_a), \text{old\_uids})$ , such that  $\text{old\_uids} = \{u : (a, u) \in S\}$ 
    - $S.A := S \cup \{(a, \text{uid}_a)\} \setminus \{(\_, u) : u \in \text{old\_uids}\} ; S.R = S.R \cup \text{old\_uids}$
  - $\text{remove } (a) \rightarrow \text{remove}( \text{old\_uids})$ , such that  $\text{old\_uids} = \{u : (a, u) \in S\}$ 
    - $S.A := S \setminus \text{old\_uids} ; S.R = S.R \cup \text{old\_uids}$
  - $\text{lookup}() : \text{returns } \{a : (a, \_) \in S\}$
- Merge ( $S_1, S_2$ ) =  $S_3$ :
  - $S_3.R = S_1.R \cup S_2.R$
  - $S_3.A = (S_1.A \cup S_2.A) \setminus \{(a, \text{uid}) : \text{uid} \in S_3.R\}$

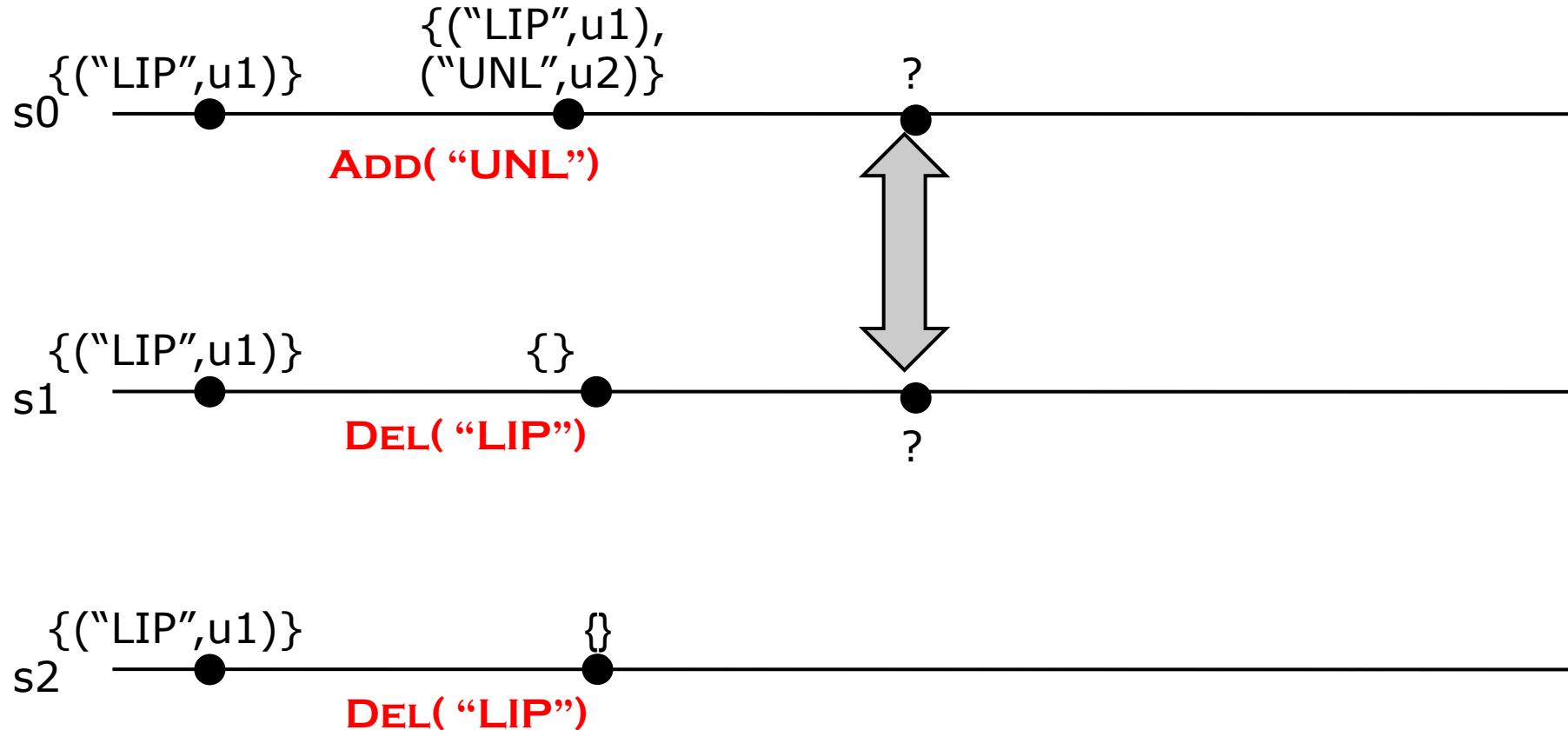
# Observed-remove Set CvRDT



# Garbage collection

- Context
  - CRDTs with operations  $\text{add}(\text{uid})$ ,  $\text{delete}(\text{uid})$ 
    - E.g. Treedoc, Logoot, OR-set, etc.
- Problem
  - On executing  $\text{DELETE}(\text{UID})$ , can we discard information on uid ?
  - What is the result of synchronization when  
 $\text{uid} \in \text{Replica1}$  and NOT  $\text{uid} \in \text{Replica2}$

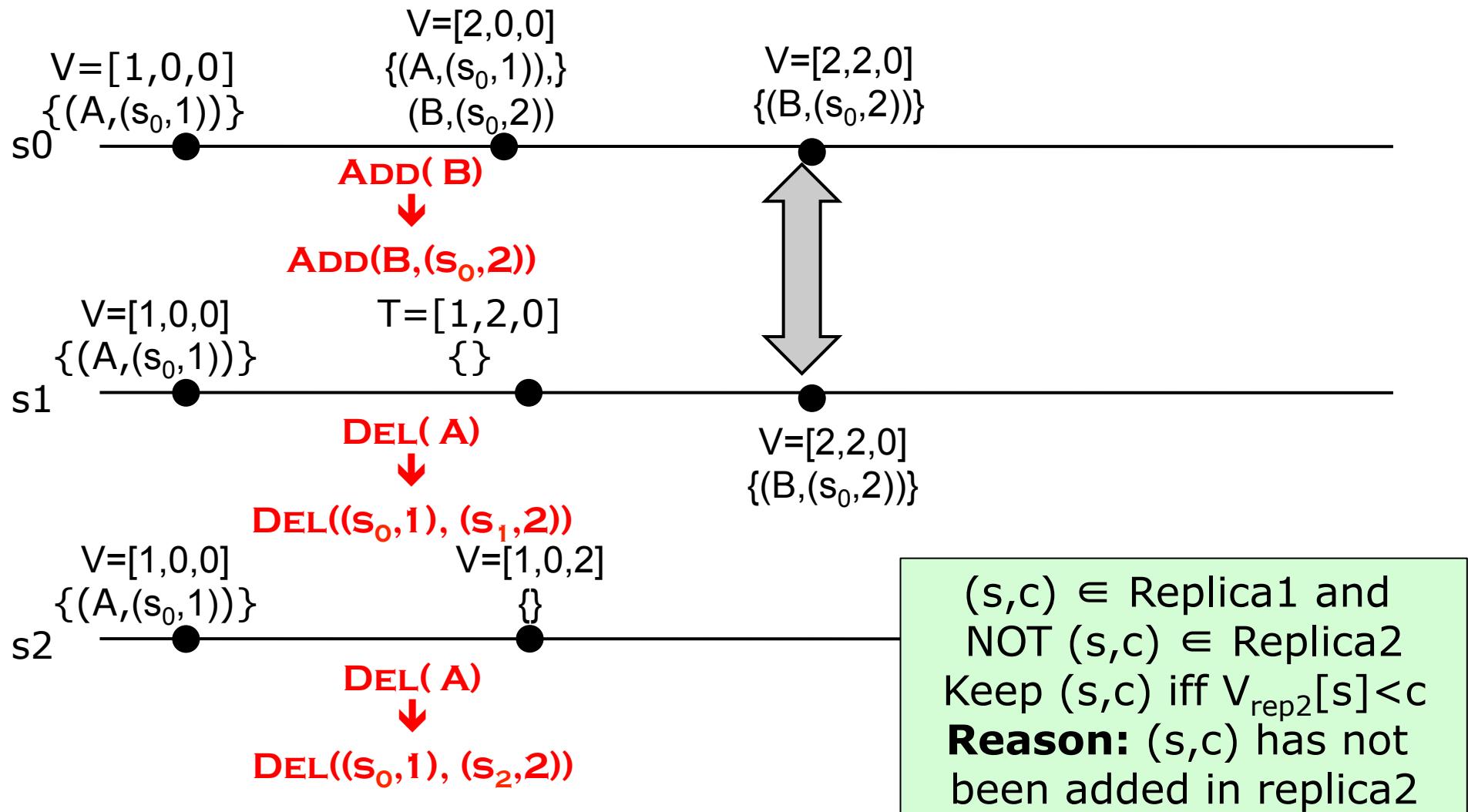
# Observed-remove Set CvRDT



# Garbage collection

- Context
  - CRDTs with operations  $\text{add}(\text{uid})$ ,  $\text{delete}(\text{uid})$ 
    - E.g. Treedoc, Logoot, OR-set, etc.
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  - On executing  $\text{DELETE}(\text{UID})$ , can we discard information on uid ?
  - What is the result of synchronization when  
 $\text{uid} \in \text{Replica1}$  and NOT  $\text{uid} \in \text{Replica2}$
- Idea
  - Generate UIDs in a way that they can be summarized in a simple way
  - Keep the summary of elements seen
- Solution
  - $\text{UID} = (\text{siteId}, \text{counter})$  Lamport timestamp
  - Summary of seen elements: version vector

# Observed-remove Set CvRDT with GC



# GC basic algorithm

- State (at site  $i$ ): version vector  $VV_i$ , set  $S_i$ , lamport clock  $c_i$
- Operations:
  - $\text{add}( ) \rightarrow \text{add}((s_i, c_i))$ 
    - $S.S := S.S \cup \{(s_i, c_i)\}; VV_i[s_i] := c_i$
  - $\text{remove}( (s, c)) \rightarrow \text{remove}( (s, c)), c_i++$ 
    - $S.S := S.S \setminus \{(s, c)\}; VV_i[s_i] := c$
- Merge (  $S_1, S_2$  ) =  $S_3$ :
  - $S_3.VV = \text{entry\_max}(S_1.VV, S_2.VV)$
  - $S_3.S = (S_1.S \wedge S_2.S) \cup \{(s, c) \in S_1.S \setminus S_2.S : S_2.VV[s] < c\} \cup \{(s, c) \in S_2.S \setminus S_1.S : S_1.VV[s] < c\}$

# Garbage-collection

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- Same approach can be used when tombstones are used
  - E.g. in any problem with operation pairs: add(uid), remove(uid)
- This includes Graphs, state-based versions of Treedoc, Logoot, etc.
- It is possible to use a single version-vector for a set of objects that are synchronized at the same time

# Need for snapshots

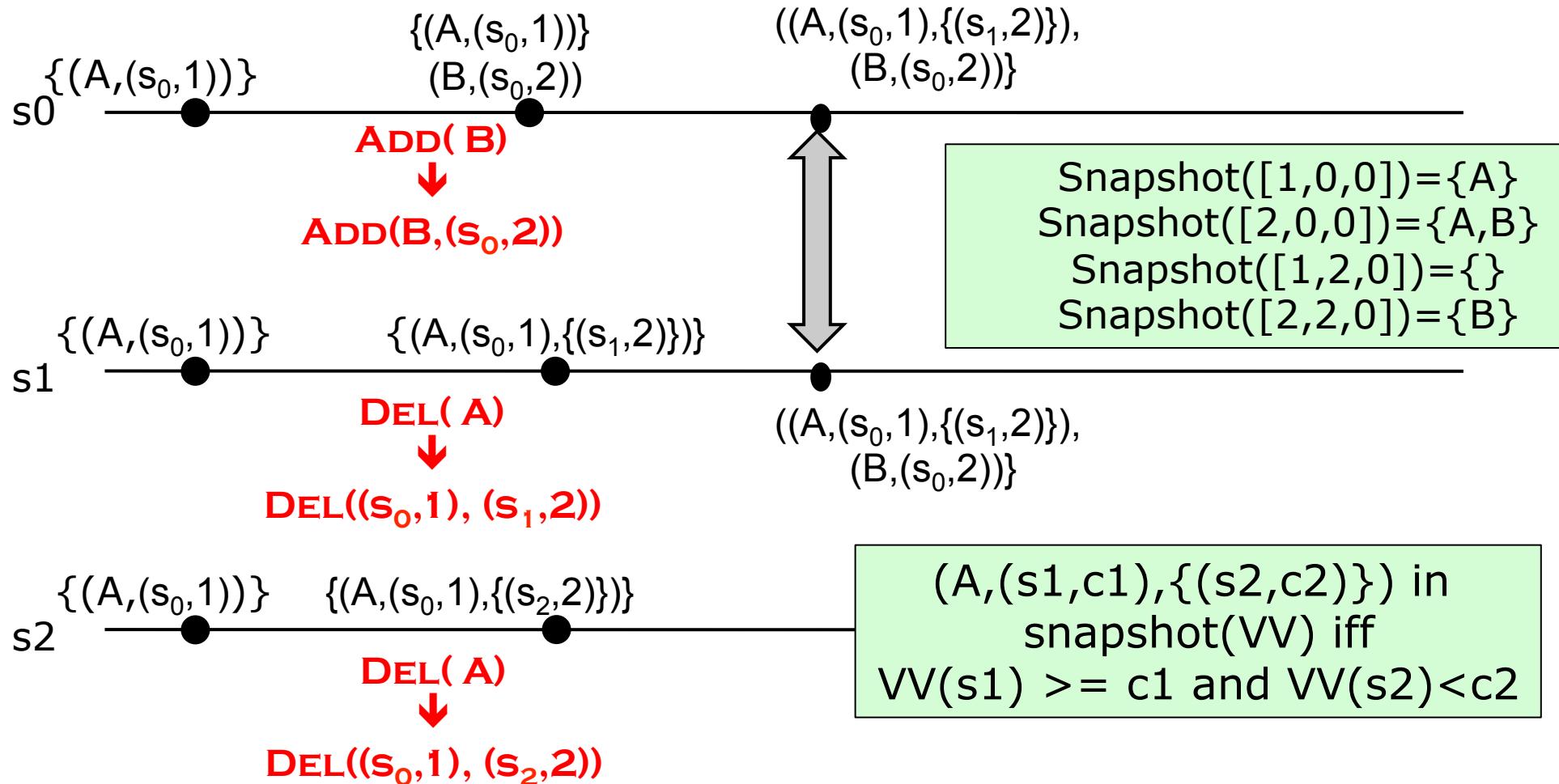
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- Some computations require accessing a consistent state of the data
  - Transactions access a consistent view of the database (in most isolation levels, at least ☺ )
- This applies to search engines
  - E.g. Google Percolator requires transactions with snapshot isolation

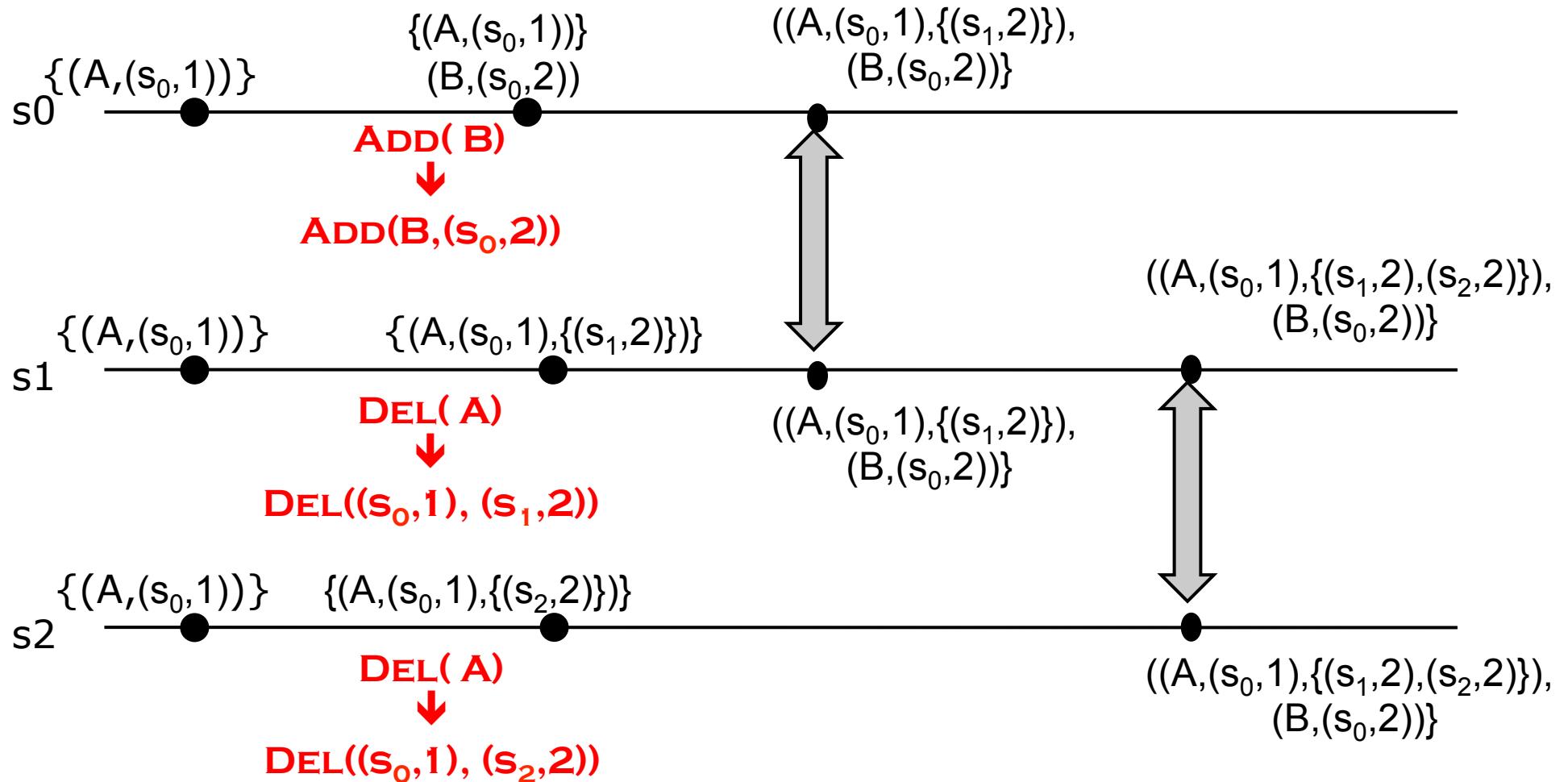
# Snapshots in CRDTs

- Context
  - CRDTs we have built include global unique identifiers (UID)
- Idea
  - Generate these UIDs in a way that they can be used to define a snapshot easily
- Solution
  - $\text{UID} = (\text{siteId}, \text{counter})$  Lamport timestamp
  - Snapshot version defined by version vector
  - Need to keep tombstones (while a snapshot is being used)

# Observed-remove Set CvRDT with snapshot



# Observed-remove Set CvRDT with snapshot



# Snapshot basic algorithm

- State (at site  $i$ ): set  $S_i$ , lamport clock  $c_i$
- Operations:
  - $\text{add}(v) \rightarrow \text{add}(v, (s_i, c_i +))$ 
    - $S.S := S.S \cup \{(v, (s_i, c_i))\}$
  - $\text{remove}(v) \rightarrow \text{remove}(v, (s, c), (s_i, c_i +))$ 
    - for each  $(s, c)$ , such that  $(v, (s, c)) \in S.S$
    - $S.S := S.S \setminus \{(v, (s, c))\} \cup \{(v, (s, c), (s_i, c_i))\}$
  - $\text{lookup}(v, VV) \rightarrow ((v, (s, c)) \in S \wedge VV[s] \geq c) \vee ((v, (s_a, c_a), (s_r, c_r)) \in S \wedge VV[s_a] \geq c_a \wedge VV[s_r] < c_r)$
- Merge (  $S_1, S_2$ ):
  - $S_3.S = S_1.S \cup S_2.S \setminus \{(v, a) \in S_1.S : (v, a, r) \in S_2.S\} \setminus \{(v, a) \in S_2.S : (v, a, r) \in S_1.S\}$

# Using snapshots in CRDTs

- Support access to consistent data snapshot in transactions
  - Information on deletions only necessary while processing transactions
  - Can be combined with GC support
- Support access to data evolution history
  - Needs to keep all information – no garbage collection
  - Added to Treedoc

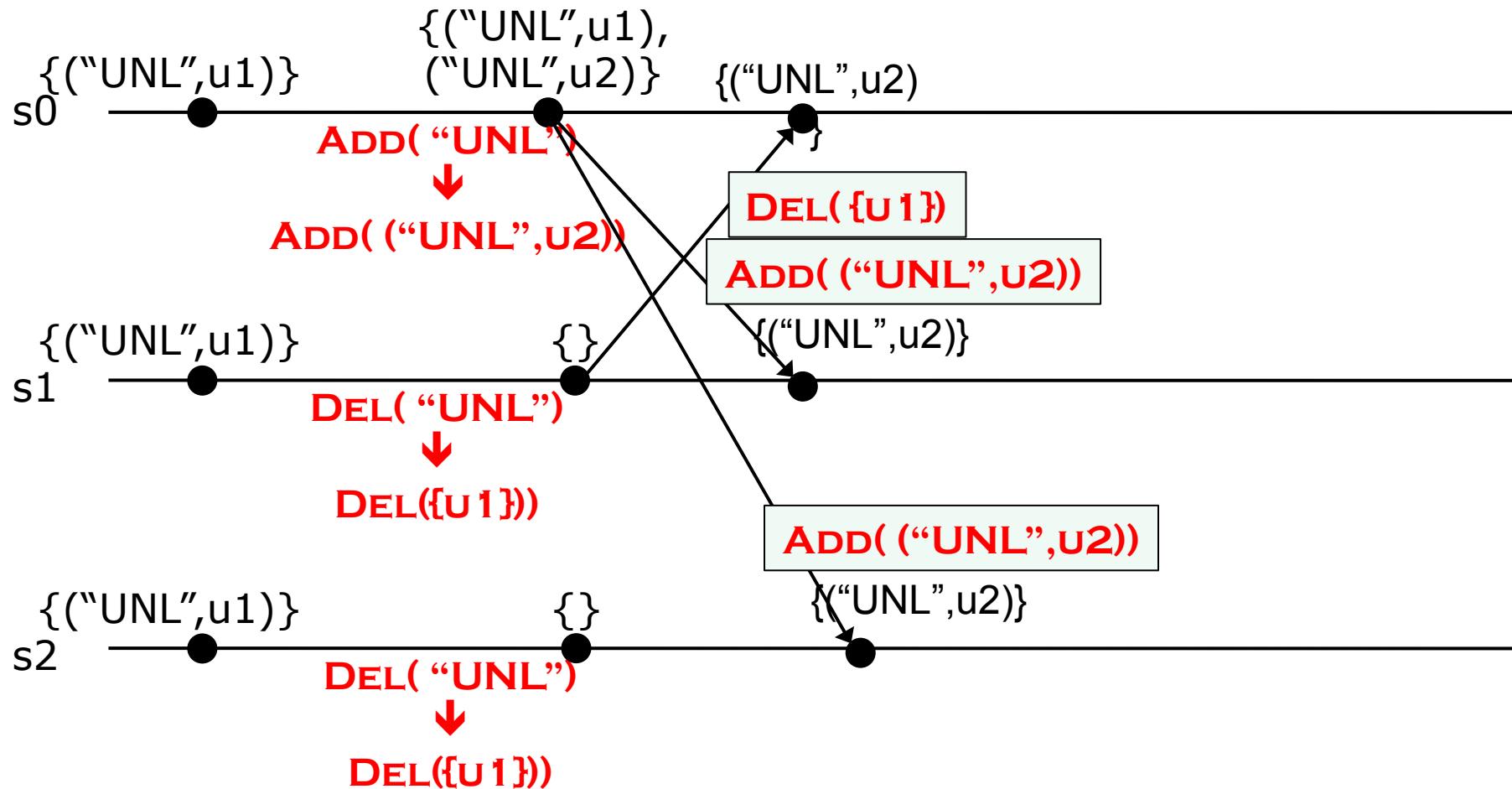
# Final remarks

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- Directed graph for web search application
  - Built using observed-remove set
- Garbage-collection optimization
  - Useful as replacement for tombstones
- Snapshot support
  - Support for transactions
  - Support for history browsing

## Backup slides

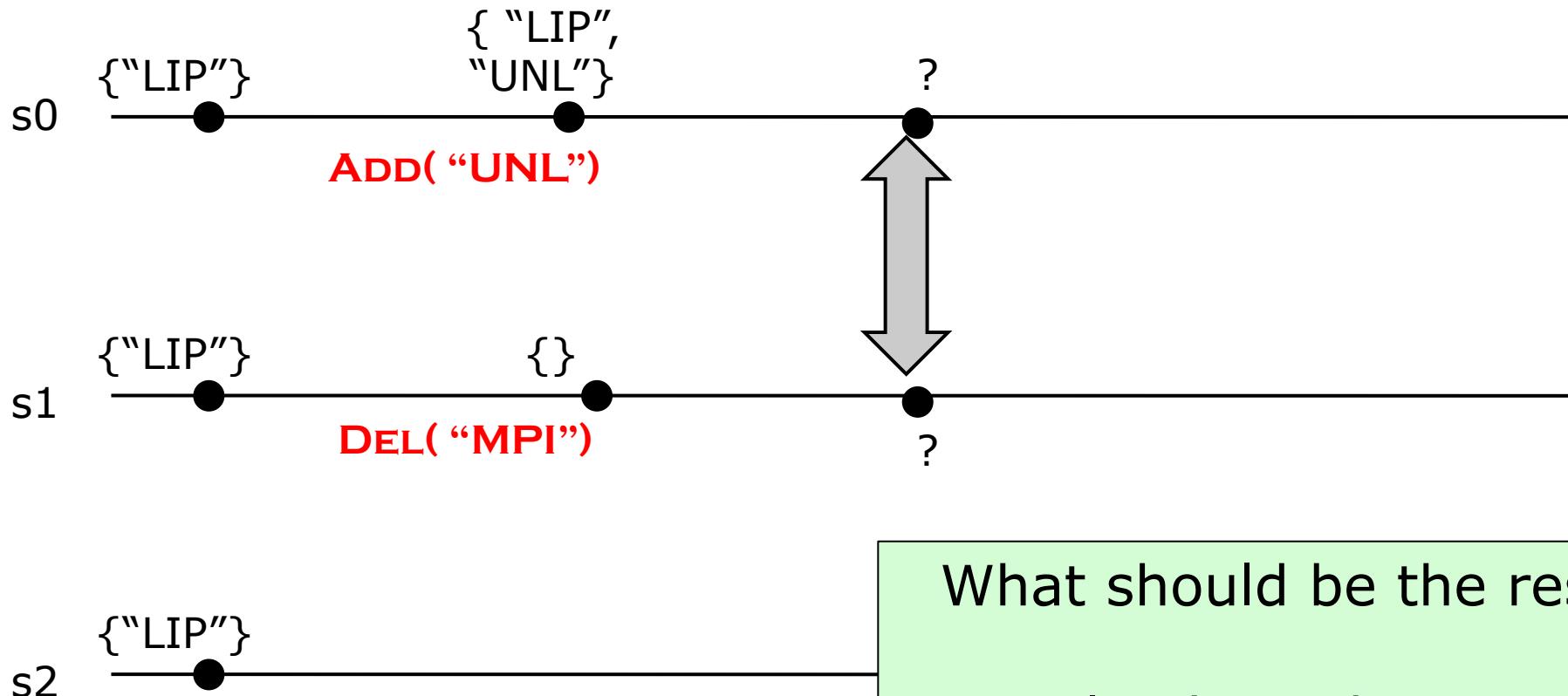
# Observed-remove Set CmRDT



# Observed-remove Set CmRDT

- State: set  $S$  of pairs  $(a, \text{uid\_}a)$
- Operations:
  - add ( $a$ ) ->  $\text{add}( (a, \text{uid\_}a), \text{old\_uids})$ , such that  $\text{old\_uids} = \{u : (a, u) \in S\}$ 
    - $S := S \cup \{(a, \text{uid\_}a)\} \setminus \{(\_, u) : u \in \text{old\_uids}\}$
  - delete( $a$ ) ->  $\text{delete}(\text{old\_uids})$ , such that  $\text{old\_uids} = \{u : (a, u) \in S\}$ 
    - $S := S \setminus \text{old\_uids}$
    - *Idea: removes all observed uids for the given atom*
  - $\text{lookup}()$  : returns  $\{a : (a, \_) \in S\}$

# Observed-remove Set CvRDT



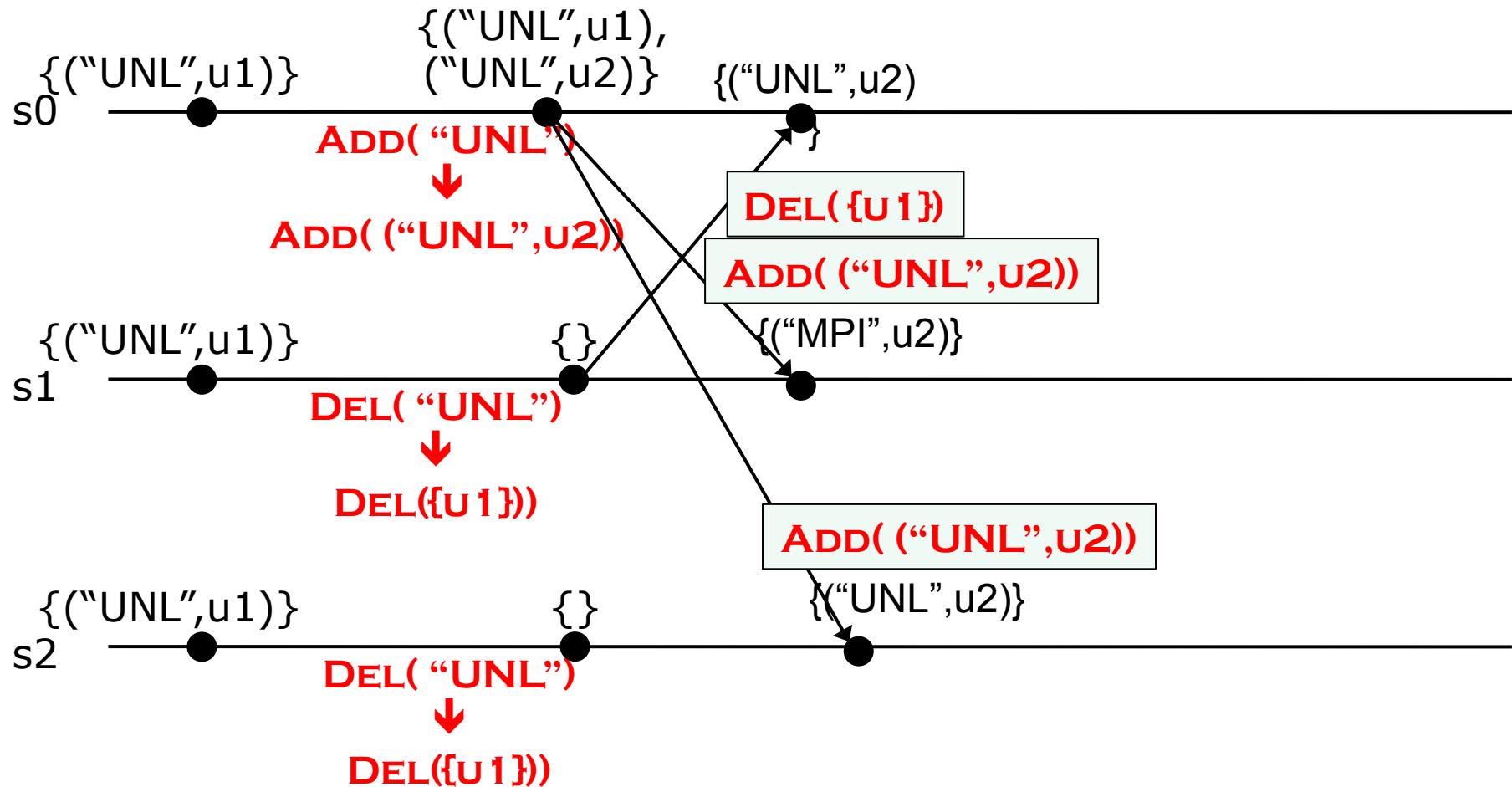
What should be the result?

Union leads to the Dynamo's shopping cart anomaly:  
 $\{ \text{"UNL"}, \text{"LIP"} \}$

# Observed-remove remove Set CRDT

- Semantics
  - Maintains a set of atoms (that are not necessarily unique)
  - On concurrent add and delete of the same atom, remove wins
- Operations:
  - Add( a ) -> adds the atom to the set
  - Delete( a ) [Pre:  $a \in \text{set}$ ] -> removes the atom from the set
  - Lookup() -> returns the atoms in the set

# Observed-remove remove Set CmRDT



# Observed-remove remove Set

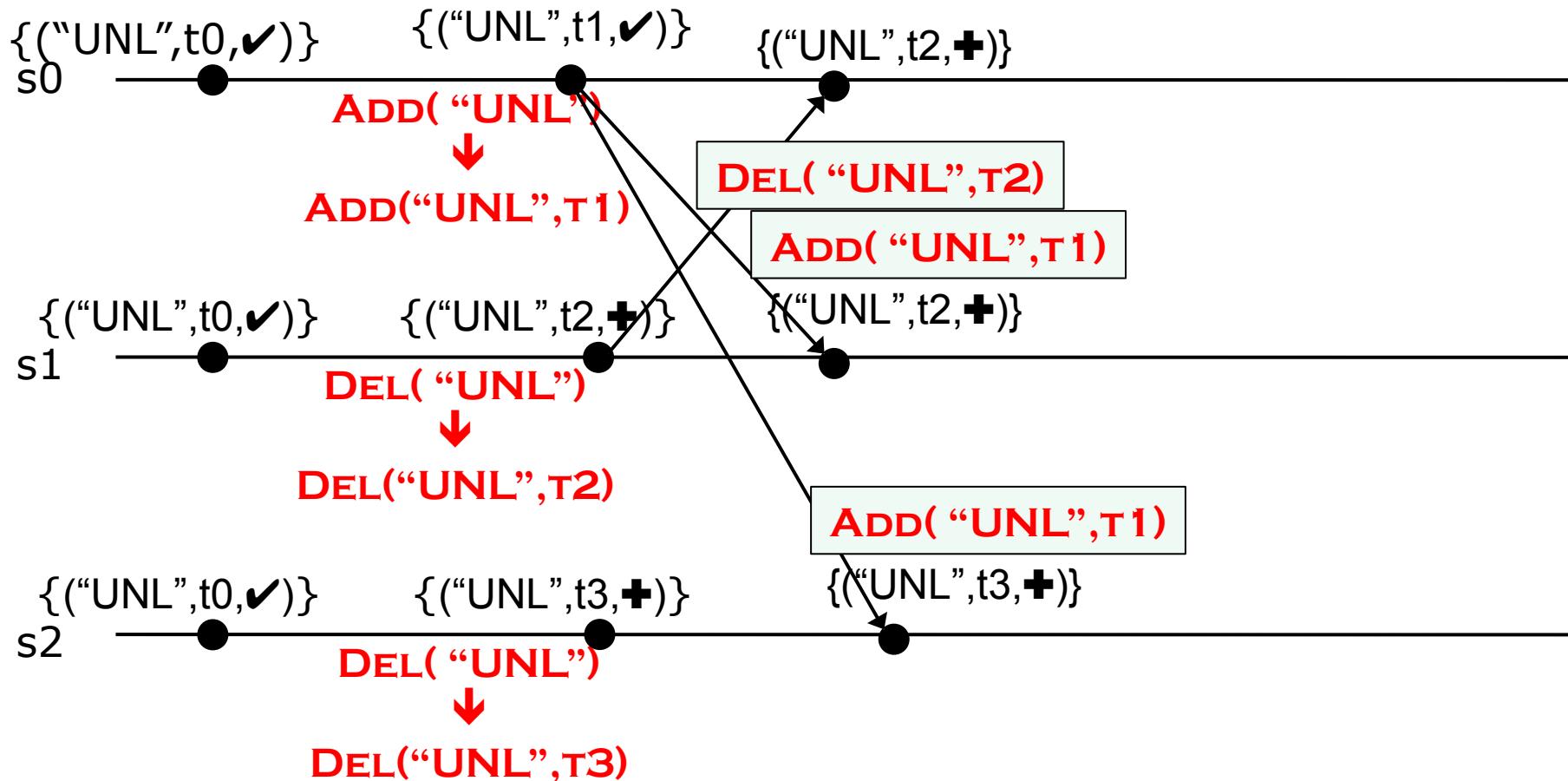
## CmRDT

- State: set  $S$  of pairs  $(a, \text{uid\_}a)$
- Operations:
  - add ( $a$ ) -> add(  $(a, \text{uid\_}a)$ ,  $\text{old\_uids}$ ), such that  $\text{old\_uids} = \{u : (a, u) \in S\}$ 
    - $S := S \cup \{(a, \text{uid\_}a)\} \setminus \{(\_, u) : u \in \text{old\_uids}\}$
  - delete( $a$ ) -> delete(  $\text{old\_uids}$  ), such that  $\text{old\_uids} = \{u : (a, u) \in S\}$ 
    - $S := S \setminus \text{old\_uids}$
    - *Idea: removes all observed uids for the given atom*
  - lookup() : returns  $\{a : (a, \_) \in S\}$

# Observed-remove LWW Set CRDT

- Semantics
  - Maintains a set of atoms (that are not necessarily unique)
  - On concurrent add and delete of the same atom, last writer operation wins
- Operations:
  - Add( a ) -> adds the atom to the set
  - Delete( a ) [Pre:  $a \in \text{set}$ ] -> removes the atom from the set
  - Lookup() -> returns the atoms in the set

# Observed-remove LWW Set CmRDT



# Observed-remove LWW Set CmRDT

- State: set  $S$  of tuples  $(a, ts, \checkmark | \oplus)$
- Operations:
  - add ( $a$ ) ->  $\text{add}( a, ts)$ , such that  $ts$  is a globally ordered clock – e.g. *Lamport clock, real-time*  
$$\begin{aligned} &\text{if } (a, ts\_old, s) \in S \text{ then} \\ &\quad \text{if } ts\_old < ts \text{ then } S = S \cup \{(a, ts, \checkmark)\} \setminus \{(a, ts\_old, s)\} \text{ endif} \\ &\quad \text{else } S = S \cup \{(a, ts, \checkmark)\} \end{aligned}$$
  - delete( $a$ ) ->  $\text{delete}( a, ts)$ , such that  $ts$  is a globally ordered clock – e.g. *Lamport clock, real-time*  
$$\begin{aligned} &\text{delete}( old\_uids) , \text{ such that } old\_uids=\{u:(a,u) \in S\} \\ &\text{if } (a, ts\_old, s) \in S \text{ then} \\ &\quad \text{if } ts\_old < ts \text{ then } S = S \cup \{(a, ts, \oplus)\} \setminus \{(a, ts\_old, s)\} \text{ endif} \\ &\quad \text{else } S = S \cup \{(a, ts, \oplus)\} \end{aligned}$$
  - $\text{lookup}()$  : returns  $\{a:(a, \_, \checkmark) \in S\}$